UNIT 3: Rules of Differentiation (Derivatives) Highlights

As a Concept: **Recall the Derivative calculates the slope of the tangent line (rate of change).**
Differentiation refers to the process of finding the derivative.
Differentiable means the value for the slope of the tangent line exists.

By definition:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Vocabulary quirks:

- ✓ Slope of the Secant Line → Traditional Slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ (Average Rate of Change)
- ✓ Slope of the Tangent Line → Use Derivative $f'(x) = ?$ (Instantaneous Rate of Change)
- ✓ Horizontal Tangent Line means “the slope equals 0!”

Algebraic Rules

- ✓ Power Rule $f(x) = x^n$ → $f'(x) = c \cdot nx^{n-1}$
- ✓ Product Rule $(f \cdot g)'(x) = f(x) \cdot g'(x) + f'(x) \cdot g(x)$
- ✓ Quotient Rule $\left(\frac{f}{g}\right)'(x) = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$ **,** $g(x) \neq 0$
- ✓ Chain Rule $fog(x) = f(g(x))$ → $fog'(x) = f'(g(x)) \bullet g'(x)$
- ✓ Implicit Derivatives Required to find $\frac{dy}{dx} = ?$ when x,y are on the same side of the equation and cannot be separated.

Trigonometric Rules

- ✓ Basic Trig Derivatives

$y = \sin x$	→	$y' = \cos x$	$y = \csc x$	→	$y' = -\csc x \cot x$
$y = \cos x$	→	$y' = -\sin x$	$y = \sec x$	→	$y' = \sec x \tan x$
$y = \tan x$	→	$y' = \sec^2 x$	$y = \cot x$	→	$y' = -\csc^2 x$

- ✓ Chain Rule

-Raised to a Power	Ex: $y = \cos^4 x$	→	$y' = -4\cos^3 x \sin x$
-Unusual Angle	Ex: $y = \sin(3x)$	→	$y' = 3\cos(3x)$
-Combination	Ex: $y = \tan^3(2x^2)$	→	$y' = 12x\tan^2(2x^2)\sec^2(2x)$

Higher Order Derivatives

Notations for Derivatives of $y = f(x)$

Derivative	f' Notation	y' Notation	D Notation	Leibniz Notation
First	$f'(x)$	y'	$D_x y$	$\frac{dy}{dx}$
Second	$f''(x)$	y''	$D_x^2 y$	$\frac{d^2y}{dx^2}$
Third	$f'''(x)$	y'''	$D_x^3 y$	$\frac{d^3y}{dx^3}$
Fourth	$f^{(4)}(x)$	$y^{(4)}$	$D_x^4 y$	$\frac{d^4y}{dx^4}$
Fifth	$f^{(5)}(x)$	$y^{(5)}$	$D_x^5 y$	$\frac{d^5y}{dx^5}$
Sixth	$f^{(6)}(x)$	$y^{(6)}$	$D_x^6 y$	$\frac{d^6y}{dx^6}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
nth	$f^{(n)}(x)$	$y^{(n)}$	$D_x^n y$	$\frac{d^ny}{dx^n}$